![NXP logo]

NOTE: The theory in this application note is still applicable, but some of the products referenced may be discontinued.

Impedance Matching Networks Applied to RF Power Transistors

By: B. Becciolini

1. INTRODUCTION

Some graphic and numerical methods of impedance matching will be reviewed here. The examples given will refer to high frequency power amplifiers.

Although matching networks normally take the form of filters and therefore are also useful to provide frequency discrimination, this aspect will only be considered as a corollary of the matching circuit.

Matching is necessary for the best possible energy transfer from stage to stage. In RF-power transistors the input impedance is of low value, decreasing as the power increases, or as the chip size becomes larger. This impedance must be matched either to a generator — of generally 50 ohms internal impedance — or to a preceding stage. Impedance transformation ratios of 10 or even 20 are not rare. Interstage matching has to be made between two complex impedances, which makes the design still more difficult, especially if matching must be accomplished over a wide frequency band.

2. DEVICE PARAMETERS

2.1 INPUT IMPEDANCE

The general shape of the input impedance of RF-power transistors is as shown in Figure 1. It is a large signal parameter, expressed here by the parallel combination of a resistance R_p and a reactance X_p (Ref. (1)).

The equivalent circuit shown in Figure 2 accounts for the behavior illustrated in Figure 1.

With the presently used stripline or flange packaging, most of the power devices for VHF low band will have their R_p and X_p values below the series resonant point fs. The input impedance will be essentially capacitive.

Most of the VHF high band transistors will have the series resonant frequency within their operating range, i.e. be purely resistive at one single frequency f_s, while the parallel resonant frequency f_p will be outside.

Parameters for one or two gigahertz transistors will be beyond f_s and approach f_p. They show a high value of R_p and X_p with inductive character.

A parameter that is very often used to judge on the broadband capabilities of a device is the input Q or Q_{IN} defined simply as the ratio R_p/X_p. Practically Q_{IN} ranges around 1 or less for VHF devices and around 5 or more for microwave transistors.
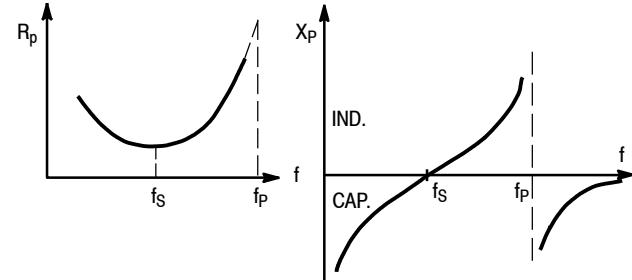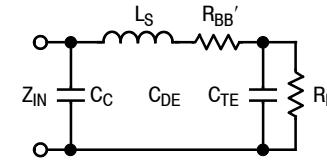

Figure 1. Input Impedance of RF-Power Transistors as a Function of Frequency

Where:
   R_E = emitter diffusion resistance
   C_{DE}, C_{TE} = diffusion and transition capacitances of the
                  emitter junction
   R_{BB}' = base spreading resistance
   C_C = package capacitance
   L_S = base lead inductance

Figure 2. Equivalent Circuit for the Input Impedance of RF-Power Transistors

Q_{IN} is an important parameter to consider for broadband matching. Matching networks normally are low-pass or pseudo low-pass filters. If Q_{IN} is high, it can be necessary to use band-pass filter type matching networks and to allow insertion losses. But broadband matching is still possible. This will be discussed later.

2.2 OUTPUT IMPEDANCE

The output impedance of the RF-power transistors, as given by all manufacturers' data sheets, generally consists of only a capacitance C_{OUT}. The internal resistance of the transistor is supposed to be much higher than the load and is normally neglected. In the case of a relatively low internal resistance, the efficiency of the device would decrease by the factor:

$$1 + R_L/R_T$$

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where R_L is the load resistance, seen at the collector-emitter terminals, and R_T the internal transistor resistance equal to:

$$\frac{1}{\omega_T, (C_{TC} + C_{DC})} \; ,$$

defined as a small signal parameter, where:

ω_T = transit angular frequency
$C_{TC} + C_{DC}$ = transition and diffusion capacitances at the collector junction

The output capacitance C_{OUT}, which is a large signal parameter, is related to the small signal parameter C_{CB}, the collector-base transition capacitance.

Since a junction capacitance varies with the applied voltage, C_{OUT} differs from C_{CB} in that it has to be averaged over the total voltage swing. For an abrupt junction and assuming certain simplifications, $C_{OUT} = 2 C_{CB}$.

Figure 3 shows the variation of C_{OUT} with frequency. C_{OUT} decreases partly due to the presence of the collector lead inductance, but mainly because of the fact that the base-emitter diode does not shut off anymore when the operating frequency approaches the transit frequency f_T.
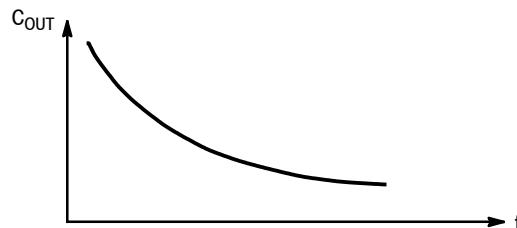

Figure 3. Output Capacitance C_{OUT} as a Function of Frequency

3. OUTPUT LOAD

In the absence of a more precise indication, the output load R_L is taken equal to:

$$R_L = \frac{[V_{CC} - V_{CE(sat)}]^2}{2P_{OUT}}$$

with $V_{CE(sat)}$ equal to 2 or 3 volts, increasing with frequency.

The above equation just expresses a well-known relation, but also shows that the load, in first approximation, is not related to the device, except for $V_{CE(sat)}$. The load value is primarily dictated by the required output power and the peak voltage; it is not matched to the output impedance of the device.

At higher frequencies this approximation becomes less exact and for microwave devices the load that must be presented to the device is indicated on the data sheet. This parameter will be measured on all Freescale RF-power devices in the future.

Strictly speaking, impedance matching is accomplished only at the input. Interstage and load matching are more impedance transformations of the device input impedance and of the load into a value R_L (sometimes with additional reactive component) that depends essentially on the power demanded and the supply voltage.

4. MATCHING NETWORKS

In the following matching networks will be described by order of complexity. These are ladder type reactance networks.

The different reactance values will be calculated and determined graphically. Increasing the number of reactances broadens the bandwidth. However, networks consisting of more than four reactances are rare. Above four reactances, the improvement is small.

4.1 NUMERICAL DESIGN

4.1.1 Two-Resistance Networks

Resistance terminations will first be considered. Figure 4 shows the reactive L-section and the terminations to be matched.
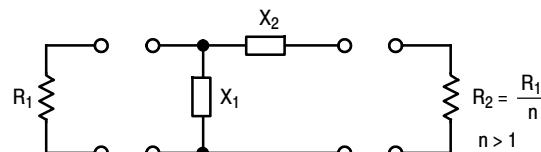

Figure 4. Two-Reactance Matching Network

Matching or exact transformation from R_2 into R_1 occurs at a single frequency f_o.

At f_o, X_1 and X_2 are equal to:

$$X_1 = \pm\, R_1 \sqrt{\frac{R_2}{R_1 - R_2}} = R_1 \frac{1}{\sqrt{n - 1}}$$

$$X_2 = \mp \sqrt{R_2 (R_1 - R_2)} = R_1 \frac{\sqrt{n - 1}}{1}$$

At f_o: $X_1 \cdot X_2 = R_1 \cdot R_2$

X_1 and X_2 must be of opposite sign. The shunt reactance is in parallel with the larger resistance.

The frequency response of the L-section is shown in Figure 5, where the normalized current is plotted as a function of the normalized frequency.
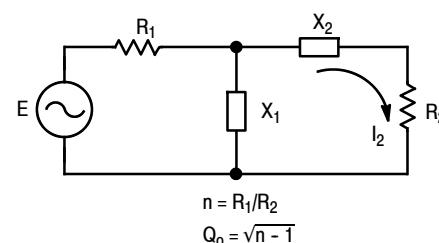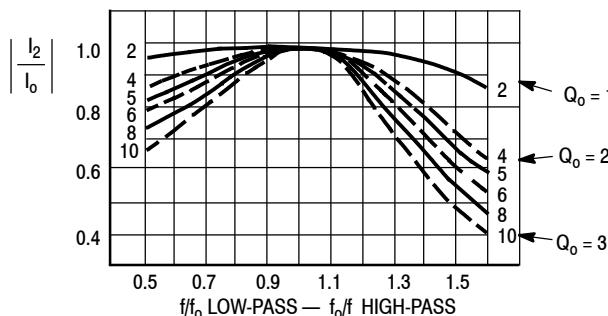

Figure 5. Normalized Frequency Response for the L-Section in Low-Pass or High-Pass Form

If X_1 is capacitive and consequently X_2 inductive, then:

$$X_1 = -\frac{f_o}{f} \, R_1 \, \sqrt{\frac{R_2}{R_1 - R_2}} = -\frac{f_o}{f} \, R_1 \, \frac{1}{\sqrt{n-1}}$$

and $\quad X_2 = \frac{f}{f_o} \sqrt{R_2 (R_1 - R_2)} = \frac{f}{f_o} \, R_1 \, \frac{\sqrt{n-1}}{1}$

The normalized current absolute value is equal to:

$$\left| \frac{I_2}{I_o} \right| = \frac{2\sqrt{n}}{\sqrt{(n-1)^2 \cdot \left(\frac{f}{f_o}\right)^4 - 2\left(\frac{f}{f_o}\right)^2 + (n+1)^2}}$$

where $I_o = \dfrac{\sqrt{n}E}{2 \cdot R_1}$, and is plotted in Figure 5 (Ref. (2)).

If X_1 is inductive and consequently X_2 capacitive, the only change required is a replacement of f by f_o and vice-versa. The L-section has low pass form in the first case and high-pass form in the second case.

The Q of the circuit at f_o is equal to:

$$Q_o = \frac{X_2}{R_2} = \frac{R_1}{X_1} = \sqrt{n-1}$$

For a given transformation ratio n, there is only one possible value of Q. On the other hand, there are two symmetrical solutions for the network, that can be either a low-pass filter or a high-pass filter.

The frequency f_o does not need to be the center frequency, $(f_1 + f_2)/2$, of the desired band limited by f_1 and f_2.

In fact, as can be seen from the low-pass configuration of Figure 5, it may be interesting to shift f_o toward the high band edge frequency f_2 to obtain a larger bandwidth w, where

$$w = \frac{2 (f_1 + f_2)}{f_2 - f_1}$$

This will, however, be at the expense of poorer harmonic rejection.

Example:

For a transformation ratio n = 4, it can be determined from the above relations:

Bandwidth w	0.1	0.3
Max insertion losses	0.025	0.2
X_1/R_1	1.730	1.712

If the terminations R_1 and R_2 have a reactive component X, the latter may be taken as part of the external reactance as shown in Figure 6.

This compensation is applicable as long as

$$Q_{INT} = \frac{X_{INT}}{R_2} \quad or \quad \frac{R_1}{X_{INT}} \; < n - 1$$

Tables giving reactance values can be found in Ref. (3) and (4).
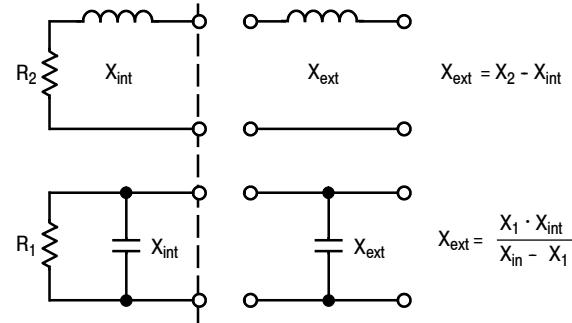

Figure 6. Termination Reactance Compensation

4.1.1.1 Use of transmission lines and inductors

In the preceding section, the inductance was expected to be realized by a lumped element. A transmission line can be used instead (Fig. 7).
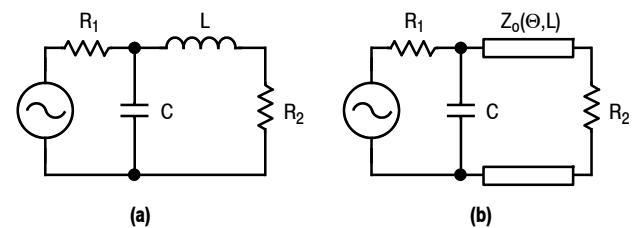

Figure 7. Use of a Transmission Line in the L-Section

As can be seen from the computed selectivity curves (Fig. 8) for the two configurations, transmission lines result in a larger bandwidth. The gain is important for a transmission line having a length $L = \lambda/4$ ($\Theta = 90°$) and a characteristic impedance $Z_o = \sqrt{R_1 \cdot R_2}$. It is not significant for lines short with respect to $\lambda/4$. One will notice that there is an infinity of solutions, one for each value of C, when using transmission lines.

4.1.2 Three-Reactance Matching Networks

The networks which will be investigated are shown in Figure 9. They are made of three reactances alternatively connected in series and shunt.

A three-reactances configuration allows to make the quality factor Q of the circuit and the transformation ratio n = R_2/R_1 independent of each other and consequently to choose the selectivity between certain limits.

For narrow band designs, one can use the following formulas (Ref. (5) AN267, where tables are given):

Network (a):

$$X_{C1} = R_1/Q \qquad \text{Q must be first selected}$$

$$X_{C2} = R_2 \sqrt{\frac{R_1/R_2}{(Q^2 + 1) - \dfrac{R_1}{R_2}}}$$

$$X_L = \frac{QR_1 + (R_1 R_2/X_{C2})}{Q^2 + 1}$$
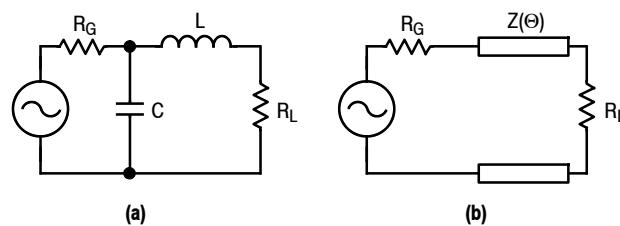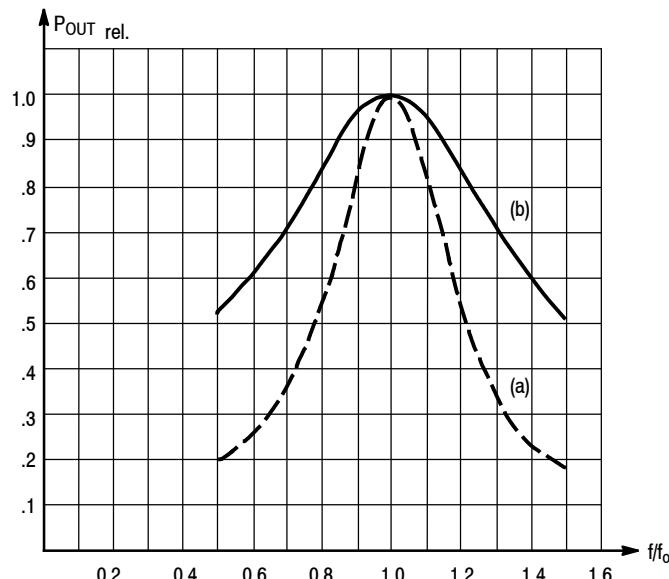
Figure 8. Bandwidth of the L-Section for n = 10
(a) with Lumped Constants
**(b) with a Transmission Line (λ/4)**
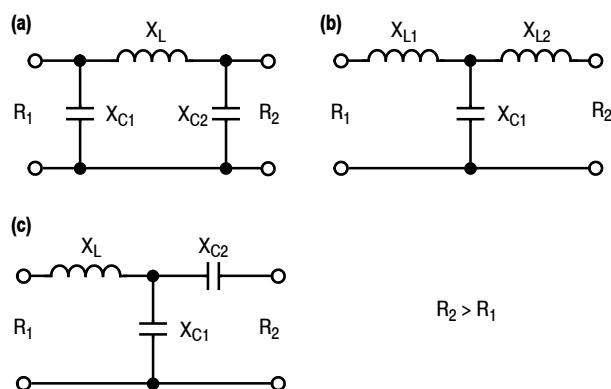
Figure 9. Three-Reactance Matching Networks

Network (b):

$$X_{L1} = R_1 Q \qquad \text{Q must be first selected}$$

$$X_{L2} = R_2 \cdot B \qquad A = R_1 (1 + Q^2)$$

$$X_{C1} = \frac{A}{Q + B} \qquad B = \sqrt{\frac{A}{R_2} - 1}$$

Network (c):

$$X_{L1} = Q \cdot R_1 \qquad \qquad \text{Q must be first selected}$$

$$X_{C2} = A \cdot R_2 \qquad \qquad A = \sqrt{\frac{R_1 (1 + Q^2)}{R_2} - 1}$$

$$X_{C1} = \frac{B}{Q - A} \qquad \qquad B = R_1 \cdot (1 + Q^2)$$

The network which yields the most practical component values, should be selected for a given application.

The three-reactance networks can be thought of as being formed of a L-section (two reactances) and of a compensation reactance. The L-section essentially performs the impedance transformation, while the additional reactance compensates for the reactive part of the transformed impedance over a certain frequency band.

Figure 10 shows a representation in the Z-plane of the circuit of Figure 9 (a) split into two parts $R_1 - C_1 - L_1$ and $C_2 - R_2$.

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$$Z' = \frac{R_1}{1 + \omega^2 R_1{}^2 C_1{}^2} + j\omega \left[L1 - \frac{R_1{}^2 C_1}{1 + \omega^2 R_1{}^2 C_1{}^2} \right] = R' + jX'$$

$$Z'' = \frac{R_2}{1 + \omega^2 C_2{}^2 R_2{}^2} - \frac{j\omega^2 C_2 R_2{}^2}{1 + \omega^2 C_2{}^2 R_2{}^2} = R'' + jX''$$

$$|x'| = \left(\frac{L_1}{C_1 R_1{}^2} - \frac{R'}{R_1} \right) \sqrt{\frac{R_1}{R'} - 1}$$

$$|x''| = \sqrt{R' R_2 - R'^2}$$
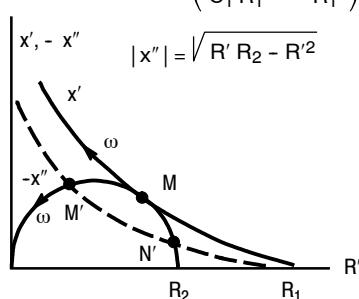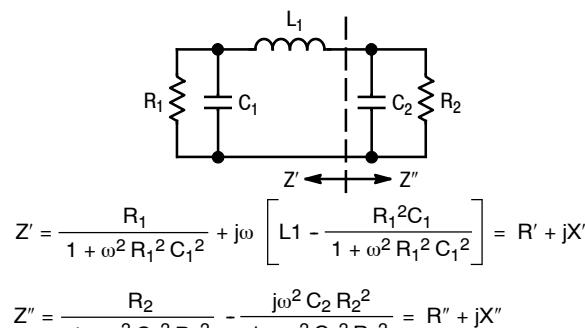
Figure 10. Z-Plane Representation of the Circuit of Figure 9(a)

Exact transformation from R_1 into R_2 occurs at the points of intersection M and N. Impedances are then conjugate or $Z' = R' + jX'$ and $Z'' = R'' + jX''$ with $R' = R''$ and $X' = -X''$.

The only possible solution is obtained when X' and $-X''$ are tangential to each other. For the dashed curve, representing another value of L_1 or C_1, a wider frequency band could be expected at the expense of some ripple inside the band. However, this can only be reached with four reactances as will be shown in section 4.1.3.

With a three-reactance configuration, there are not enough degrees of freedom to permit $X' = -X''$ and simultaneously obtain the same variation of frequency on both curves from M' to point N'.

Exact transformation can, therefore, only be obtained at one frequency.

The values of the three reactances can be calculated by making

$$X' = -X'', \quad R' = R'' \text{ and } \frac{dX'}{dR'} = -\frac{dX''}{dR''} \quad .$$
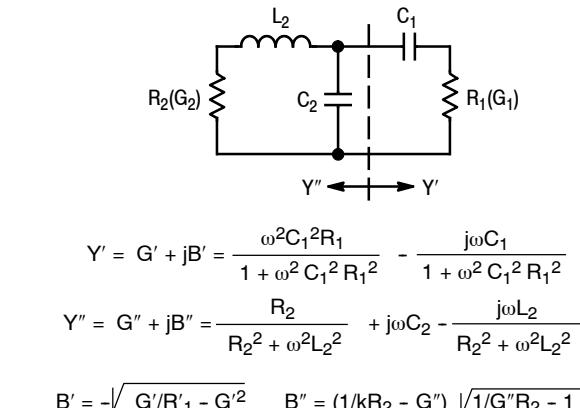
The general solution of these equations leads to complicated calculations. Therefore, computed tables should be used.

One will note on Figure 10 that the compensation reactance contributes somewhat to impedance transformation, i.e. R' varies when going from M to R_2.

The circuit of Figure 9 (b) is dual with respect to the first one and gives exactly the same results in a Y-plane representation.

Circuit of Figure 9 (c) is somewhat different since only one intersection M exists as shown in Figure 11. Narrower frequency bands must be expected from this configuration. The widest band is obtained for $C_1 = \infty$.

Again, if one of the terminations has a reactive component, the latter can be taken as a part of the matching network, provided it is not too large (see Fig. 6).


$$Y' = G' + jB' = \frac{\omega^2 C_1{}^2 R_1}{1 + \omega^2 C_1{}^2 R_1{}^2} - \frac{j\omega C_1}{1 + \omega^2 C_1{}^2 R_1{}^2}$$

$$Y'' = G'' + jB'' = \frac{R_2}{R_2{}^2 + \omega^2 L_2{}^2} + j\omega C_2 - \frac{j\omega L_2}{R_2{}^2 + \omega^2 L_2{}^2}$$

$$B' = -\sqrt{G'/R'_1 - G'^2} \qquad B'' = (1/kR_2 - G'') \sqrt{1/G''R_2 - 1}$$

$$k = L_2/C_2 R_2{}^2$$
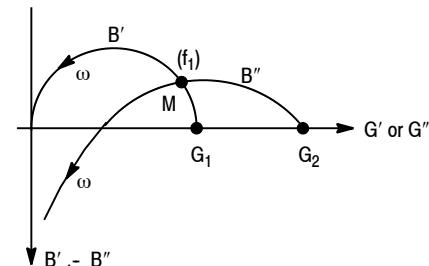
Figure 11. Y-Plane Representation of the Circuit of Figure 9(c)

4.1.3 Four-Reactance Networks

Four-reactance networks are used essentially for broadband matching. The networks which will be considered in the following consist of two two-reactance sections in cascade. Some networks have pseudo low-pass filter character, others band-pass filter character. In principle, the former show narrower bandwidth since they extend the impedance transformation to very low frequencies unnecessarily, while the latter insure good matching over a wide frequency band around the center frequency only (see Fig. 14).
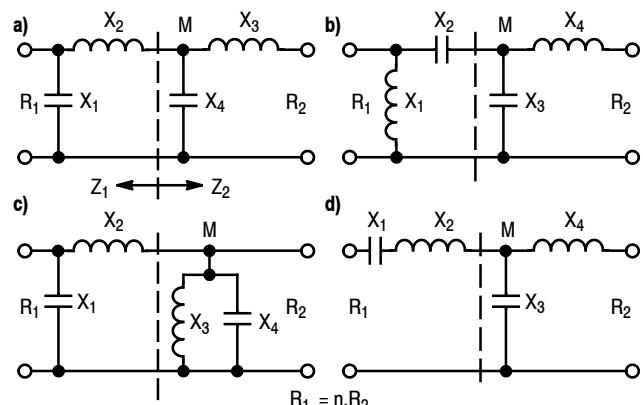

Figure 12. Four-Reactance Networks

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The two-reactance sections used in above networks have either transformation properties or compensation properties. Impedance transformation is obtained with one series reactance and one shunt reactance. Compensation is made with both reactances in series or in shunt.

If two cascaded transformation networks are used, transformation is accomplished partly by each one.

With four-reactance networks there are two frequencies, f_1 and f_2, at which the transformation from R_1 into R_2 is exact. These frequencies may also coincide.

For network (b) for instance, at point M, R_1 or R_2 is transformed into $\sqrt{R_1 R_2}$ when both frequencies fall together. At all points (M), Z_1 and Z_2 are conjugate if the transformation is exact.

In the case of Figure 12 (b) the reactances are easily, calculated for equal frequencies:

$$X_1 = \frac{R_1}{\sqrt{\sqrt{n}-1}} \quad , \quad X_2 = R_1 \sqrt{\frac{\sqrt{n}-1}{n}}$$

$$X_1 \cdot X_4 = R_1 \cdot R_2 = X_2 \cdot X_3$$

$$X_3 = \frac{R_1}{\sqrt{n(\sqrt{n}-1)}} \quad , \quad X_4 = \frac{R_1}{n}\sqrt{\sqrt{n}-1}$$

For network (a) normally, at point (M), Z_1 and Z_2 are complex. This pseudo low-pass filter has been computed elsewhere (Ref. (3)). Many tables can be found in the literature for networks of four and more reactances having Tchebyscheff character or maximally-flat response (Ref. (3), (4) and (6)).

Figure 13 shows the transformation path from R_1 to R_2 for networks (a) and (b) on a Smith-Chart (refer also to section 4.2, Graphic Design).

Case (a) has been calculated using tables mentioned in Ref. (4).

Case (b) has been obtained from the relationship given above for $X_1 \ldots X_4$. Both apply to a transformation ratio equal to 10 and for $R_1 = 1$.

There is no simple relationship for $X'_1 \ldots X'_4$ of network (b) if f_1 is made different from f_2 for larger bandwidth.

Figure 14 shows the respective bandwidths of network (a) and (b) for the circuits shown in Figure 13.

If the terminations contain a reactive component, the computed values for X_1 or X_4 may be adjusted to compensate for this.

For configuration (a), it can be seen from Figure 13, that in the considered case the Q's are equal to 1.6.

For configuration (b) Q'_1, which is equal to Q'_2, is fixed for each transformation ratio.

n	2	4	8	10	16
$Q'_1 = Q'_2$	0.65	1	1.35	1.46	1.73
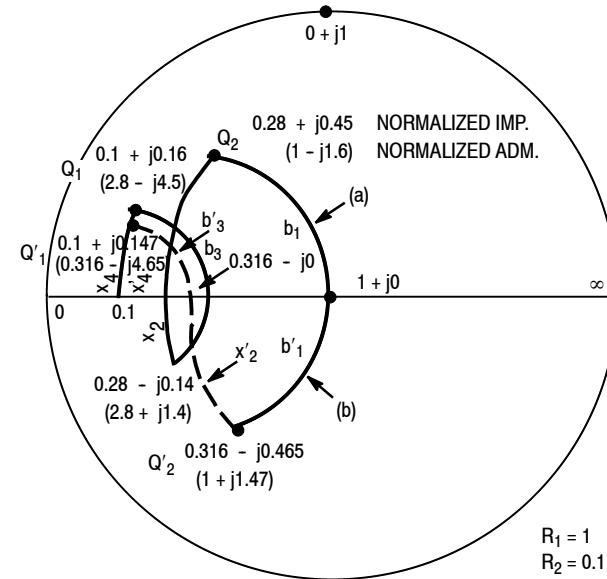
$$Q' = \sqrt{\sqrt{n}-1}$$

The maximum value of reactance that the terminations may have for use in this configuration can be determined from the above values of Q'.

If R_1 is the load resistance of a transistor, the internal transistor resistance may not be equal to R_1. In this case

the selectivity curve will be different from the curves given in Figure 14. Figure 15 shows the selectivity for networks (a) and (b) when the source resistance R_1 is infinite.

From Figure 15 it can be seen that network (a) is more sensitive to R_1 changes than network (b).

| (a)	$X_1 = 0.624$	$X_3 = 0.169$	$X'_1 = 0.68$	$X'_3 = 0.215$
	(	(	(	(
	$B_1 = 1.6$	$B_3 = 5.9$	$B'_1 = 1.47$	$B'_3 = 4.65$
	$X_2 = 0.59$	$X_4 = 0.160$	$X'_2 = 0.465$	$X'_4 = 0.147$

$Q_1 = 1.6 = Q_2$ $Q'_1 = Q'_2 = 1.47$
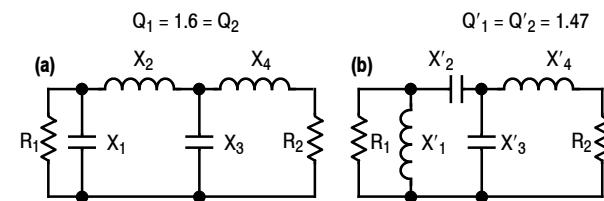
Figure 13. Transformation Paths for Networks (a) and (b)
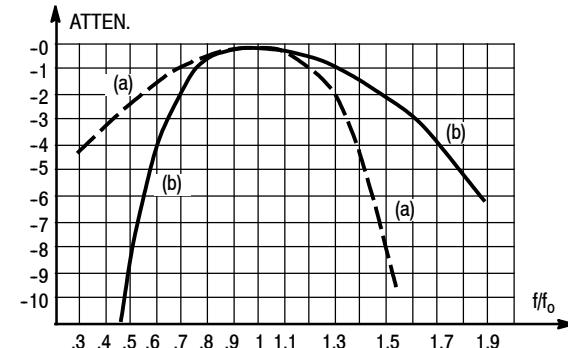
Figure 14. Selectivity Curves for Networks (a) and (b) of Figure 13

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Figure 15 graph with axes REL. ATTEN. (from 2 to -10) vs f/f₀ (.4 .5 .6 .7 .8 .9 1 1.1 1.3 1.5 1.7), curves labeled (a) and (b).
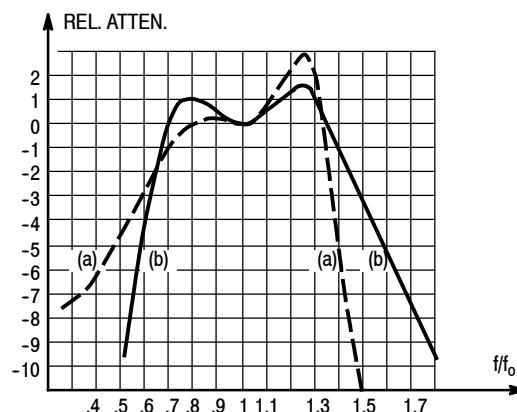
Figure 15. Selectivity Curves for Networks (a) and (b) of Figure 13 with Infinite R_1

As mentioned earlier, the four-reactance network can also be thought of as two cascaded two-reactance sections; one used for transformation, the other for compensation. Figure 16 shows commonly used compensation networks, together with the associated L-section.

The circuit of Figure 16 (a) can be compared to the three-reactance network shown in Figure 9(c). The difference is that capacitor C_2 of that circuit has been replaced by a L-C circuit. The resulting improvement may be seen by comparing Figure 17 with Figure 11.

COMPENSATION NETWORK
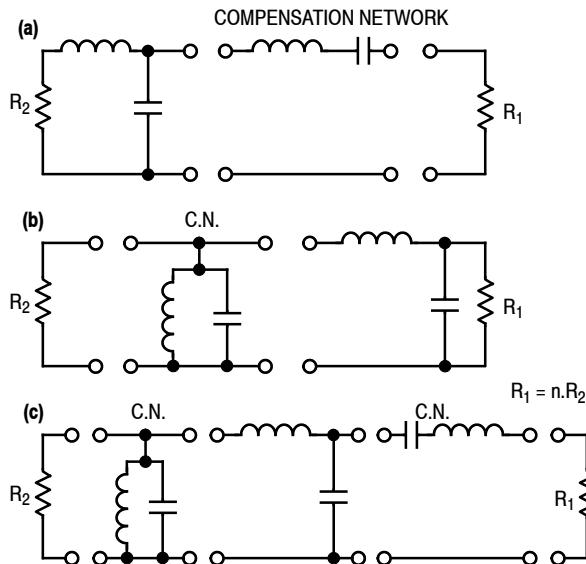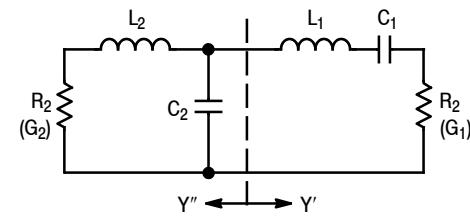
Figure 16. Compensation Networks Used with a L-Section

By adding one reactance, exact impedance transformation is achieved at two frequencies. It is now possible to choose component values such that the point of intersection M' occurs at the same frequency f_1 on both curves and simultaneously that N' occurs at the same frequency f_2 on both curves. Among the infinite number of possible intersections, only one allows to achieve this.

$$Y' = G' + jB' = \frac{R_1}{R_1^2 + (\omega L_1 - 1/\omega c_1)^2} - \frac{j(\omega L_1 - 1/\omega c_1)}{R_1^2 + (\omega L_1 - 1/\omega c_1)^2}$$

$$Y'' = G'' + jB'' = \frac{R_2}{R_2^2 + \omega^2 L_2^2} + j\omega\left(C_2 - \frac{L_2}{R_2^2 + \omega^2 L_2^2}\right)$$

$$B' = \pm\left|\sqrt{\frac{G'}{R_1} - G'^2}\right|$$

$$B'' = (1/kR_2 - G'')\left|\sqrt{\frac{1}{G''R_2} - 1}\right| \qquad k = \frac{L_2}{C_2 R_2^2}$$
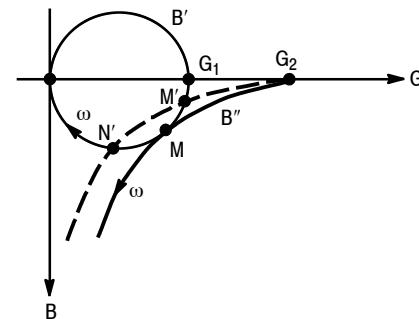
Figure 17. Y-Plane Representation of the Circuit of Figure 16(a)

When M' and N' coincide in, M, the new $dX'/df = dX''/df$ condition can be added to the condition $X' = - X''$ (for three-networks) and similarly $R' = R''$ and $dR'/df = dR''/df$.

If f_1 is made different from f_2, a larger bandwidth can be achieved at the expense of some ripple inside the band.

Again, a general solution of the above equations leads to still more complicated calculations than in the case of three-reactance networks. Therefore, tables are preferable (Ref. (3), (4) and (6)).

The circuit of Figure 16(b) is dual of the circuit of Figure 14(a) and does not need to be treated separately. It gives exactly the same results in the Z-plane. Figure 16(c) shows a higher order compensation requiring six reactive elements.

The above discussed matching networks employing compensation circuits result in narrower bandwidths than the former solutions (see paragraph 4.1.3) using two transformation sections. A matching with higher order compensation such as in Figure 16(c) is not recommended. Better use can be made of the large number of reactive elements using them all for transformation.

When the above configurations are realized using short portions of transmission lines, the equations or the usual tables no longer apply. The calculations must be carried out on a computer, due to the complexity. However, a graphic method can be used (see next section) which will consist essentially in tracing a transformation path on the Z-Y-chart using the computed lumped element values and replacing it by the closest path obtained with distributed constants. The

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bandwidth change is not significant as long as short portions of lines are used (Ref. (13)).

4.1.4 Matching Networks Using Quarter-Wave Transformers

At sufficiently high frequencies, where $\lambda/4$-long lines of practical size can be realized, broadband transformation can easily be accomplished by the use of one or more $\lambda/4$-sections.

Figure 18 summarizes the main relations for (a) one-section and (b) two-section transformation.

A compensation network can be realized using a $\lambda/2$-long transmission line.

(a)
$R_1 = nR_2$

$Z = \sqrt{R_1 R_2} = R_2 \sqrt{n} = \dfrac{R_1}{\sqrt{n}}$

(b)
$Z_1 = \sqrt[4]{R_1{}^3 R_2}$

$Z_2 = \sqrt[4]{R_1 R_2{}^3}$

at A, $R = \sqrt{R_1 R_2}$
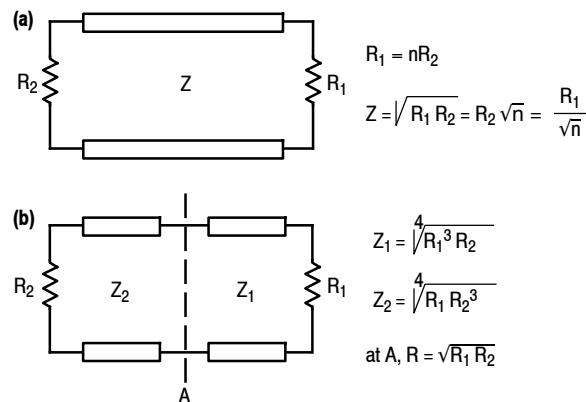
Figure 18. Transformation Networks Using $\lambda/4$-Long Transmission Lines

Figures 19 and 20 show the selectivity curves for different transformation ratios and section numbers.

Exponential lines

Exponential lines have largely frequency independent transformation properties.

The characteristic impedance of such lines varies exponentially with their length I:

$$Z = Z_o \cdot e^{kl}$$

where k is a constant, but these properties are preserved only if k is small.
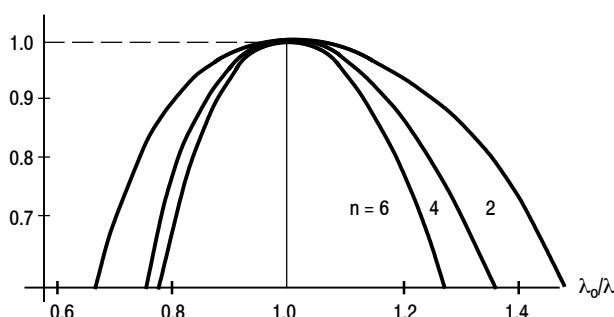
Figure 19. Selectivity Curves for Two $\lambda/4$-Section Networks at Different Transformation Ratios
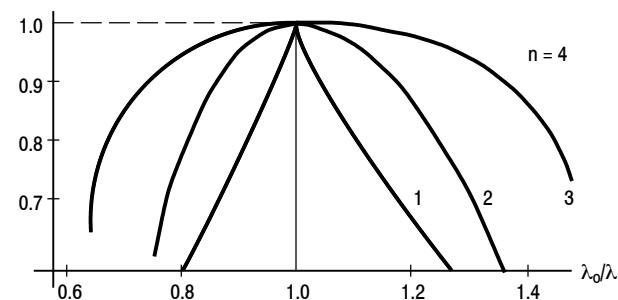
Figure 20. Selectivity Curves for One, Two and Three $\lambda/4$-Sections

4.1.5 Broadband Matching Using Band-Pass Filter Type Networks. High Q Case.

The above circuits are applicable to devices having low input or output Q, if broadband matching is required. Generally, if the impedances to be matched can be represented for instance by a resistor R in series with an inductor L (sometimes a capacitor C) within the band of interest and if L is sufficiently low, the latter can be incorporated into the first inductor of the matching network. This is also valid if the representation consists of a shunt combination of a resistor and a reactance.

Practically this is feasible for Q's around one or two. For higher Q's or for input impedances consisting of a series or parallel resonant circuit (see Fig. 2), as it appears to be for large bandwidths, a different treatment must be followed.

Let us first recall that, as shown by Bode and Fano (Ref. (7) and (8)), limitations exist on the impedance matching of a complex load. In the example of Figure 21, the load to be matched consists of a capacitor C and a resistor R in shunt.
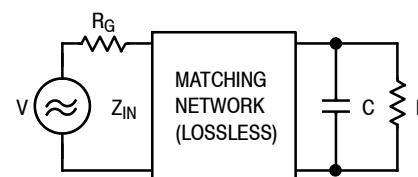

Figure 21. General Matching Conditions

The reflection coefficient between transformed load and generator is equal to:

$$\Gamma = \dfrac{Z_{IN} - R_g}{Z_{IN} + R_g}$$

$\Gamma = 0$, perfect matching,
$\Gamma = 1$, total reflection.

The ratio of reflected to incident power is:

$$\dfrac{Pr}{Pi} = |\Gamma|^2$$

The fundamental limitation on the matching takes the form:

$$\int_{\omega = o}^{\infty} \ln\left(\dfrac{1}{|\Gamma|}\right) d\omega \le \dfrac{\pi}{RC} \qquad \text{Bode equation}$$

and is represented in Figure 22.

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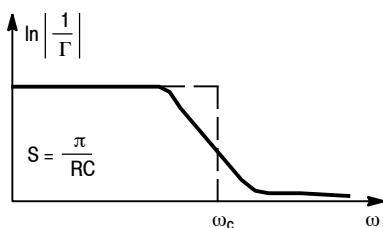
Figure 22.   Representation of Bode Equation

The meaning of Bode equation is that the area S under the curve cannot be greater than π/RC and therefore, if matching is required over a certain bandwidth, this can only be done at the expense of less power transfer within the band. Thus, power transfer and bandwidth appear as interchangeable quantities.

It is evident that the best utilization of the area S is obtained when $|\Gamma|$ is kept constant over the desired band ω_c and made equal to 1 over the rest of the spectrum. Then

$$|\Gamma| = e^{-\dfrac{\pi}{\omega_c\,RC}}$$

within the band and no power transfer happens outside.

A network fulfilling this requirement cannot be obtained in practice as an infinite number of reactive elements would be necessary.

If the attenuation a is plotted versus the frequency for practical cases, one may expect to have curves like the ones shown in Figure 23 for a low-pass filter having Tchebyscheff character.
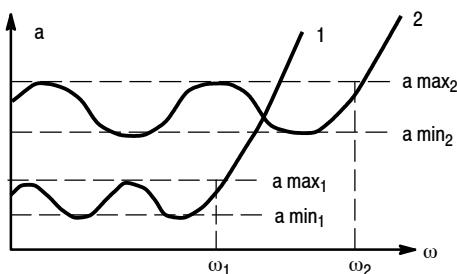
Figure 23.   Attenuation versus Angular Frequency for Different Bandwidths with Same Load

For a given complex load, an extension of the bandwidth from ω_1 to ω_2 is possible only with a simultaneous increase of the attenuation a. This is especially noticeable for Q's exceeding one or two (see Figure 24).
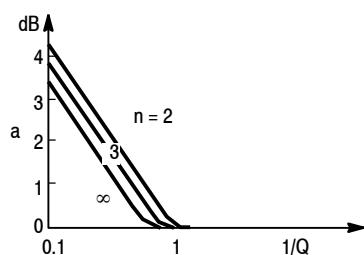
Figure 24.   Insertion Losses as a Function of 1/Q

Thus, devices having relatively high input Q's are usable for broadband operation, provided the consequent higher attenuation or reflection introduced is acceptable.

The general shape of the average insertion losses or attenuation a (neglecting the ripple) of a low-pass impedance matching network is represented in Figure 24 as a function of 1/Q for different numbers of network elements n (ref. (3)).

For a given Q and given ripple, the attenuation decreases if the number n of the network element increases. But above n = 4, the improvement is small.

For a given attenuation a and bandwidth, the larger n the smaller the ripple.

For a given attenuation and ripple, the larger n the larger the bandwidth.

Computations show that for Q < 1 and n ≤ 3 the attenuation is below 0.1 db approximately. The impedance transformation ratio is not free here. The network is a true low-pass filter. For a given load, the optimum generator impedance will result from the computation.

Before impedance transformation is introduced, a conversion of the low-pass prototype into a band-pass filter type network must be made. Figure 25 summarizes the main relations for this conversion.

r is the conversion factor.

For the band-pass filter, Q_{IN} max or the maximum possible input Q of a device to be matched, has been increased by the factor r (from Figure 25, $Q'_{IN\ max} = r \cdot Q_{IN\ max}$).

Impedance inverters will be used for impedance transformation. These networks are suitable for insertion into a band-pass filter without affecting the transmission characteristics.
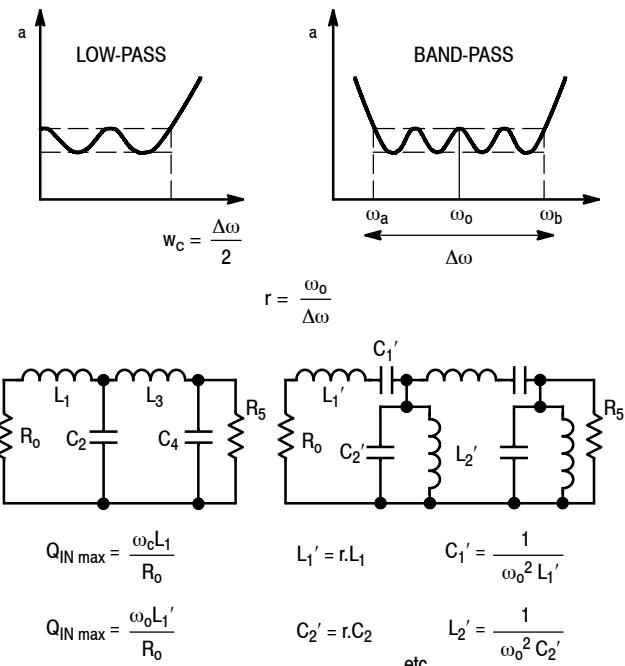

Figure 25.   Conversion from Low-Pass into Band-Pass Filter

Figure 26 shows four impedance inverters. It will be noticed that one of the reactances is negative and must be combined in the band-pass network with a reactance of at

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least equal positive value. Insertion of the inverter can be made at any convenient place (Ref. (3) and (9)).
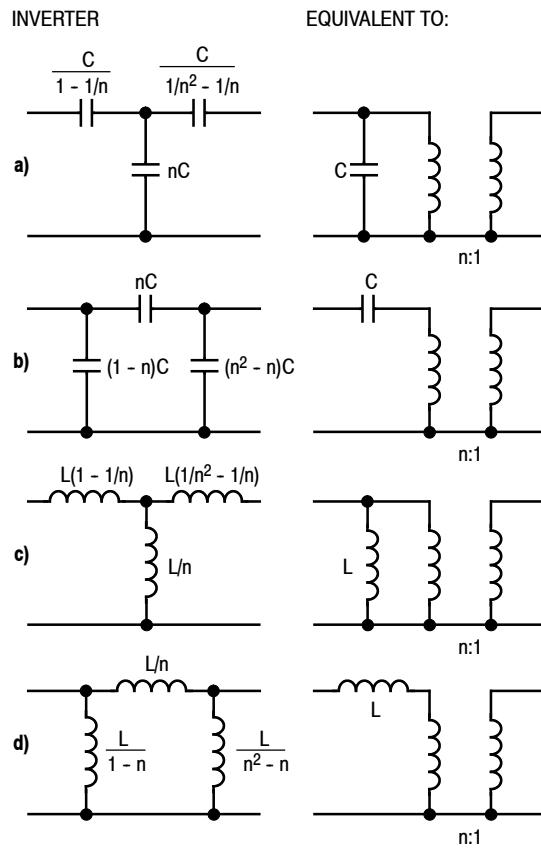

Figure 26. Impedance Inverters

When using the band-pass filter for matching the input impedance of a transistor, reactances L'_1 C'_1 should be made to resonate at ω_o by addition of a convenient series reactance.

As stated above, the series combination of Ro, L'_1 and C'_1 normally constitutes the equivalent input network of a transistor when considered over a large bandwidth. This is a good approximation up to about 500 MHz.

In practice the normal procedure for using a bandpass filter type matching network will be the following:

(1) For a given bandwidth center frequency and input impedance of a device to be matched e.g. to 50 ohms, first determine Q_{IN}' from the data sheet as

$$\frac{\omega_o L'_1}{Ro}$$

after having eventually added a series reactor for centering,

(2) Convert the equivalent circuit $R_o L_1'$ C_1' into a low-pass prototype R_o L_1 and calculate Q_{IN} using the formulas of Figure 25,

(3) Determine the other reactance values from tables (Ref. (3)) for the desired bandwidth,

(4) Convert the element values found by step (3) into series or parallel resonant circuit parameters,

(5) Insert the impedance inverter in any convenient place.

In the above discussions, the gain roll-off has not been taken into account. This is of normal use for moderate bandwidths (30% for ex.). However, several methods can be employed to obtain a constant gain within the band despite the intrinsic gain decrease of a transistor with frequency.

Tables have been computed elsewhere (Ref. (10)) for matching networks approximating 6 db/octave attenuation versus frequency.

Another method consists in using the above mentioned network and then to add a compensation circuit as shown for example in Figure 27.


$R = Z_{IN} = $ CONSTANT vs. f.
$L_1 C_1 = L_2 C_2$
$R_1 = R$
$\omega_b^2 = 1/L_1 C_1 = 1/L_2 C_2$
(HIGH BAND EDGE)
$Q_2 = \omega_b L_2/R = Q_1 = \omega_b C_1 R$
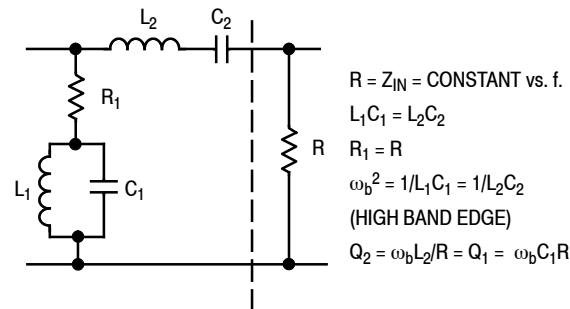
Figure 27. Roll-Off Compensation Network

Resonance ω_b is placed at the high edge of the frequency band. Choosing Q correctly, roll-off can be made 6 db/octave.

The response of the circuit shown in Figure 27 is expressed by:

$$\frac{1}{1 + Q^2 \left(\dfrac{\omega}{\omega_b} - \dfrac{\omega_b}{\omega} \right)^2} \quad , \quad \text{where } \omega < \omega_b$$

This must be equal to ω/ω_b for 6 db/octave compensation.

At the other band edge a, exact compensation can be obtained if:

$$Q = \frac{\left(\dfrac{\omega_b}{\omega_a} \right)^2 - 1}{\dfrac{\omega_a}{\omega_b} - \left(\dfrac{\omega_b}{\omega_a} \right)^2}$$

4.1.6 Line Transformers

The broadband properties of line transformers make them very useful in the design of broadband impedance matching networks (Ref. (11) and (12)).

A very common form is shown by Figure 28. This is a 4:1 impedance transformer. Other transformation ratios like 9:1 or 16:1 are also often used but will not be considered here.

The high frequency cut-off is determined by the length of line which is usually chosen smaller than λ min/8. Short lines extend the high frequency performance.

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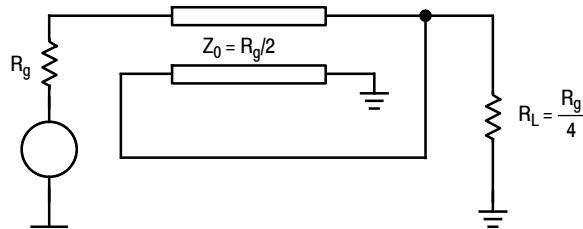
Figure 28. 4:1 Line Transformer

The low frequency cut-off is determined first by the length of line, long lines extending the low frequency performance of the transformer. Low frequency cut-off is also improved by a high even mode impedance, which can be achieved by the use of ferrite material. With matched ends, no power is coupled through the ferrite which cannot saturate.

For matched impedances, the high frequency attenuation a of the 4:1 transformer is given by:

$$a = \frac{(1 + 3 \cos 2\pi l/\lambda)^2 + 4 \sin^2 2\pi l/\lambda}{4 (1 + \cos 2\pi l/\lambda)^2}$$
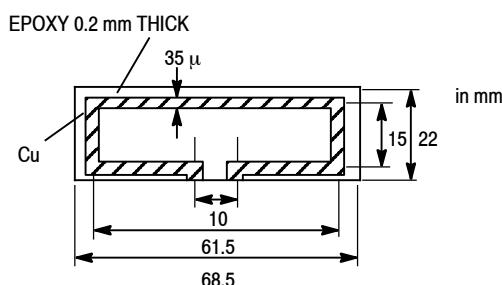
For $l = \lambda/4$, $a = 1.25$ or 1 db; for $l = \lambda/2$, $a = \infty$.

The characteristic impedance of the line transformer must be equal to:

$$Zo = \sqrt{Rg \cdot R} \quad .$$

Figures 29 and 30 show two different realizations of 4:1 transformers for a 50 to 12.5 ohm-transformation designed for the band 118 – 136 MHz.

The transformers are made of two printed circuit boards or two ribbons stuck together and connected as shown in Figures 29 and 30.
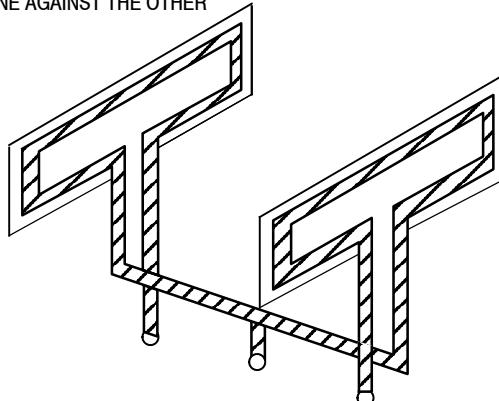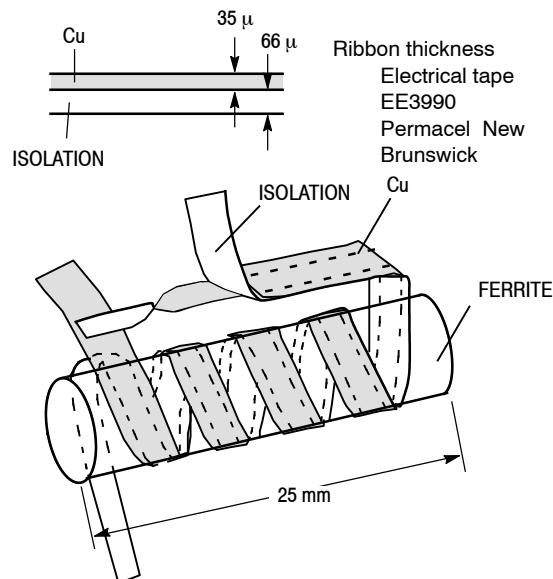
Figure 29. 4:1 Line Transformer on P.C. Board

Line Transformer on P.C. Board (Figure 29)

– Stick one ribbon (1.5 mm wide) against the other (2.5)
   Total length per ribbon – 9 cm.
   Turns 3.5

Figure 30. 4:1 Copper Ribbon Line Transformer with Ferrite

Line Transformer on Ferrite (Figure 30)

4.2 GRAPHIC DESIGN

The common method of graphic design makes use of the Impedance-Admittance Chart (Smith Chart).

It is applicable to all ladder-type networks as encountered in matching circuits.

Matching is supposed to be realized by the successive algebraic addition of reactances (or susceptances) to a given start impedance (or admittance) until another end impedance (or admittance) is reached.

Impedance chart and admittance chart can be superimposed and used alternatively due to the fact that an immittance point, defined by its reflection coefficient Γ with respect to a reference, is common to the Z-chart and the Y-chart, both being representations in the Γ-plane.

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$$\Gamma = \frac{Z - R_s}{Z + R_s} \quad \gamma_r + j\gamma_i$$

$$\Gamma = \frac{G_s - Y}{G_s + Y}$$
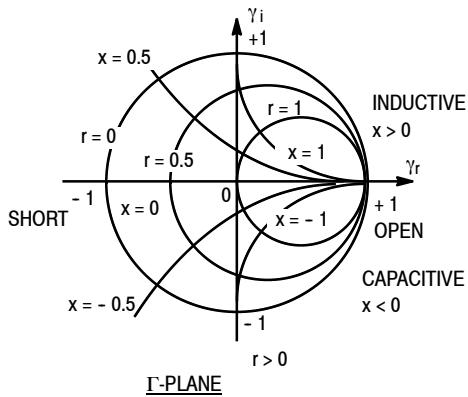
$$R_s = \frac{1}{G_s} = \text{Characteristic impedance of the line}$$

More precisely, the Z-chart is a plot in the Γ-plane, while the Y-chart is a plot in the $-\Gamma$-plane. The change from the Γ to $-\Gamma$-plane is accounted for in the construction rules given below.

Figure 31 and 32 show the representation of normalized Z and Y respectively, in the Γ-plane.

The Z-chart is used for the algebraic addition of series reactances. The Y-chart is used for the algebraic addition of shunt reactances.

For the practical use of the charts, it is convenient to make the design on transparent paper and then place it on a usual Smith-chart of impedance type (for example). For the addition of a series reactor, the chart will be placed with "short" to the left. For the addition of a shunt reactor, it will be rotated by 180° with "short" (always in terms of impedance) to the right.

$$\frac{Z}{R_s} = z = r + jx$$

$$r = \frac{1 - \gamma_r^2 - \gamma_i^2}{(1 - \gamma_r)^2 + \gamma_i^2}$$

$$x = \frac{2\gamma_i}{(1 - \gamma_r)^2 + \gamma_i^2}$$

$$\gamma_r = \frac{r^2 + x^2 - 1}{(r + 1)^2 + x^2}$$

$$\gamma_i = \frac{2x}{(r + 1)^2 + x^2}$$

r circles $\begin{cases} \text{Radius: } 1/(1 + r) \\ \text{Center at: } \gamma_r = \dfrac{r}{1 + r}, \ \gamma_i = 0 \end{cases}$

x circles $\begin{cases} \text{Radius: } 1/x \\ \text{Center at: } \gamma_r = 1, \ \gamma_i = 1/x \end{cases}$
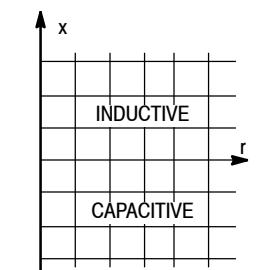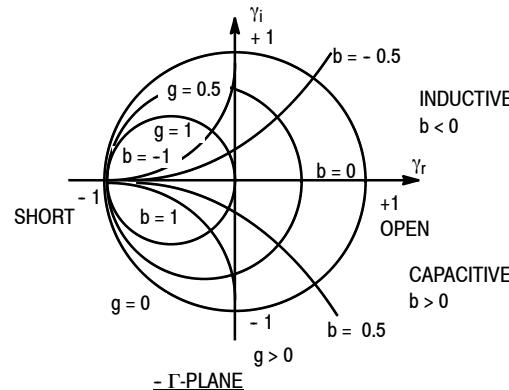
**Figure 31. Representation of the Normalized Z Values in the Γ-Plane**
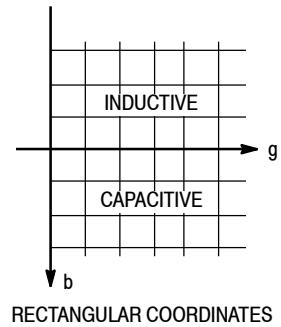
$$\frac{Y}{G_s} = y = g + jb$$

$$g = \frac{1 - \gamma_r^2 - \gamma_i^2}{(1 + \gamma_r)^2 + \gamma_i^2}$$

$$b = \frac{-2\gamma_i}{(1 + \gamma_r)^2 + \gamma_i^2}$$

$$\gamma_r = \frac{1 - g^2 - b^2}{(1 + g)^2 + b^2}$$

$$\gamma_i = \frac{-2b}{(1 + g)^2 + b^2}$$

g circles $\begin{cases} \text{Radius: } 1/(1 + g) \\ \text{Center at: } \gamma_r = -g/(1 + g); \ \gamma_i = 0 \end{cases}$

b circles $\begin{cases} \text{Radius: } 1/b \\ \text{Center at: } \gamma_r = -1, \ \gamma_i = -1/b \end{cases}$

Figure 32. Representation of the Normalized Y-Values in the Γ-Plane

The following design rules apply. They can very easily be found by thinking of the more familiar Z and Y representation in rectangular coordinates.

For joining two impedance points, there are an infinity of solutions. Therefore, one must first decide on the number of reactances that will constitute the matching network. This number is related essentially to the desired bandwidth and the transformation ratio.

Addition of	Chart to be Used	Direction	Using Curve of Constant
Series R	Z	Open (in terms of admittance)	x
Series G	Y	Short (in terms of admittance)	b
Series C (+ 1/jωC)	Z	ccw	r
Shunt C (+ jωC)	Y	cw	g
Series L (+ jωL)	Z	cw	r
Shunt L (+ 1/jωL)	Y	ccw	g

Secondly, one must choose the operating Q of the circuit, which is also related to the bandwidth. Q can be defined at each circuit node as the ratio of the reactive part to the real part of the impedance at that node. The Q of the circuit, which is normally referred to, is the highest value found along the path.

Constant Q curves can be superimposed to the charts and used in conjunction with them. In the Γ-plane Q-curves are circles with a radius equal to
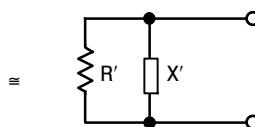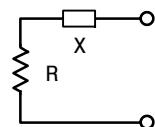
$$\sqrt{1 + \frac{1}{Q^2}}$$

and a center at the point ± 1/Q on the imaginary axis, which is expressed by:

$$Q = \frac{x}{r} = \frac{2\gamma_i}{1 - \gamma_r^2 - \gamma_i^2} \qquad \gamma_r^2 + \left(\gamma_i + \frac{1}{Q}\right)^2 = 1 + \frac{1}{Q^2}.$$

The use of the charts will be illustrated with the help of an example.

The following series shunt conversion rules also apply:

$$R = \frac{R'}{1 + \frac{R'^2}{X'^2}} \qquad\qquad G' = \frac{1}{R'} = \frac{R}{R^2 + X^2}$$

$$X = \frac{X'}{1 + \frac{X'^2}{R'^2}} \qquad\qquad -B' = \frac{1}{X'} = \frac{X}{R^2 + X^2}$$

Figure 33 shows the schematic of an amplifier using the 2N5642 RF power transistor. Matching has to be achieved at 175 MHz, on a narrow band basis.
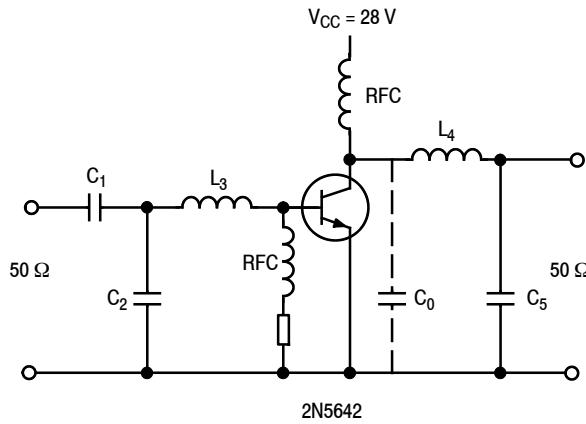
Figure 33. Narrow-Band VHF Power Amplifier

The rated output power for the device in question is 20 W at 175 MHz and 28 V collector supply. The input impedance at these conditions is equal to 2.6 ohms in parallel with – 200 pF (see data sheet). This converts to a resistance of 1.94 ohms in series with a reactance of 1.1 ohm.

The collector load must be equal to:

$$\frac{[Vcc - Vce\ (sat)]^2}{2 \times P_{out}} \qquad or \qquad \frac{(28 - 3)^2}{40} = 15.6\ ohms.$$
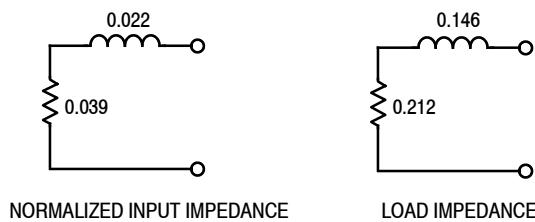
The collector capacitance given by the data sheet is 40 pF, corresponding to a capacitive reactance of 22.7 ohms.

The output impedance seen by the collector to insure the required output power and cancel out the collector capacitance must be equal to a resistance of 15.6 ohms in parallel with an inductance of 22.7 ohms. This is equivalent to a resistance of 10.6 ohms in series with an inductance of 7.3 ohms.

The input Q is equal to, 1.1/1.94 or 0.57 while the output Q is 7.3/10.6 or 0.69.

It is seen that around this frequency, the device has good broadband capabilities. Nevertheless, the matching circuit will be designed here for a narrow band application and the effective Q will be determined by the circuit itself not by the device.

Figure 34 shows the normalized impedances (to 50 ohms).

NORMALIZED INPUT IMPEDANCE   LOAD IMPEDANCE

Figure 34. Normalized Input and Output Impedances for the 2N5642

Figure 35 shows the diagram used for the graphic design of the input matching circuit. The circuit Q must be larger than about 5 in this case and has been chosen equal to 10. At Q = 5, C_1 would be infinite. The addition of a finite value of C_1 increases the circuit Q and therefore the selectivity. The normalized values between brackets in the Figure are admittances (g + jb).
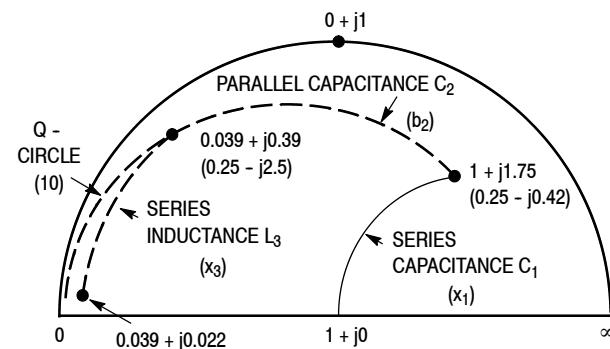

Figure 35. Input Circuit Design

At f = 175 MHz, the following results are obtained:

$$\omega L_3 = 50 x_3 = 50\ (0.39 - 0.022) = 18.5\ ohms$$
$$\therefore\quad L_3 = 16.8\ nH$$

$$\omega C_2 = \frac{1}{50}\quad b_2 = \frac{1}{50}\ (2.5 - 0.42) = 0.0416\ mhos$$

$$\therefore\quad C_2 = 37.8\ pF$$

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$$\frac{1}{\omega C_1} = 50x_1 = 50 \cdot 1.75 = 87.5 \text{ ohms}$$

$$\therefore \quad C_1 = 10.4 \text{ pF}$$

Figure 36 shows the diagram for the output circuit, designed in a similar way.
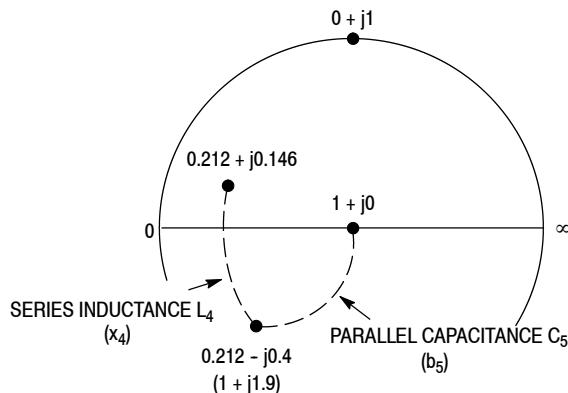
Figure 36. Output Circuit Design

Here, the results are (f = 175 MHz):

$$\omega L_4 = 50 \cdot x_4 = 50 \cdot (0.4 + 0.146) = 27.3 \text{ ohms}$$

$$\therefore \quad L_4 = 24.8 \text{ nH}$$

$$\omega C_5 = \frac{1}{50} \quad b_5 = \frac{1}{50} \cdot 1.9 = 0.038 \text{ mhos}$$

$$\therefore \quad C_5 = 34.5 \text{ pF}$$

The circuit Q at the output is equal to 1.9.

The selectivity of a matching circuit can also be determined graphically by changing the x or b values according to a chosen frequency change. The diagram will give the VSWR and the attenuation can be computed.

The graphic method is also useful for conversion from a lumped circuit design into a stripline design. The immittance circles will now have their centers on the 1 + jo point.

At low impedance levels (large circles), the difference between lumped and distributed elements is small.

5. PRACTICAL EXAMPLE

The example shown refers to a broadband amplifier stage using a 2N6083 for operation in the VHF band 118 – 136 MHz. The 2N6083 is a 12.5 V-device and, since amplitude modulation is used at these transmission frequencies, that choice supposes low level modulation associated with a feedback system for distortion compensation.

Line transformers will be used at the input and output. Therefore the matching circuits will reduce to two-reactance networks, due to the relatively low impedance transformation ratio required.

5.1 DEVICE CHARACTERISTICS

Input impedance of the 2N6083 at 125 MHz:

$$R_p = 0.9 \text{ ohms}$$
$$C_p = -390 \text{ pF}$$

Rated output power:
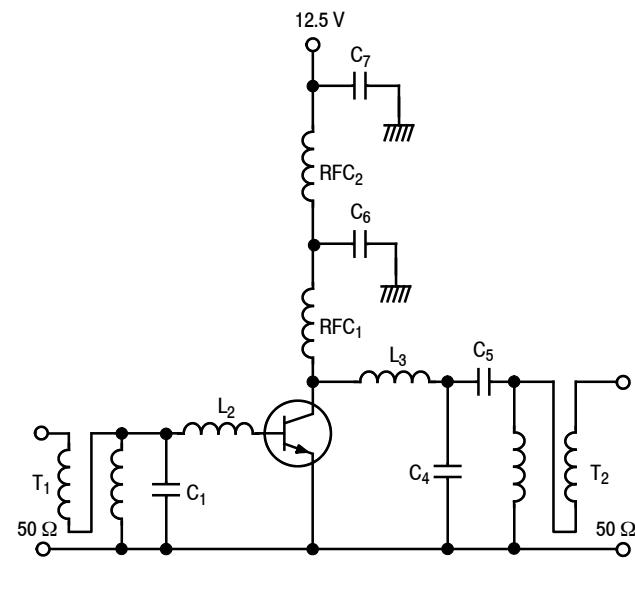
30 W for 8 W input at 175 MHz. From the data sheet it appears that at 125 MHz, 30 W output will be achieved with about 4 W input.

Output impedance:

$$\frac{[V_{cc} - V_{ce\,(sat)}]^2}{2 \times P_{out}} = \frac{100}{60} = 1.67 \text{ ohms}$$

$$C_{out} = 180 \text{ pF at } 125 \text{ MHz}$$

5.2 CIRCUIT SCHEMATIC

C$_1$ = 300 pF (chip)
L$_2$ = 4.2 nH (adjust)
L$_3$ = 8 nH (adjust)
C$_4$ = 130 pF (chip)
C$_5$ = 750 pF (chip)
C$_6$ = 2.2 μF

T$_1$, T$_2$ see Figure 30
RFC$_1$ = 7 turns
    6.3 mm coil diameter
    0.8 mm wire diameter
RFC$_2$ = 3 turns on ferrite bead
C$_7$ = 0.68 μF

Figure 37. Circuit Schematic

5.3 TEST RESULTS
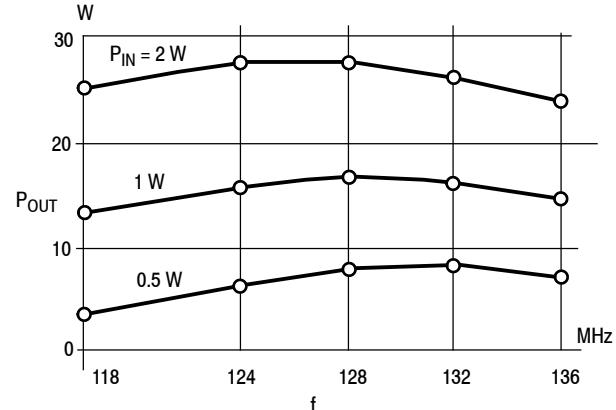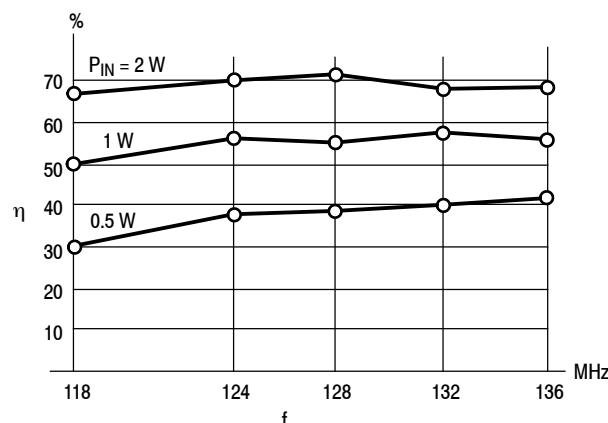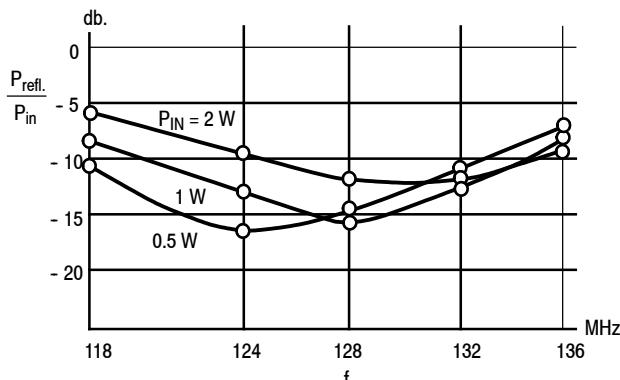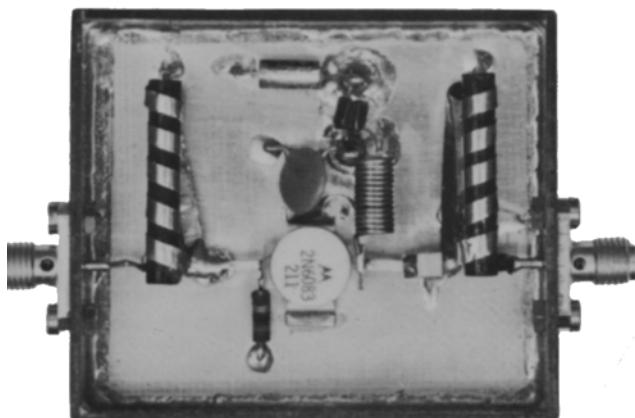

Figure 38. P$_{OUT}$ vs Frequency

**Figure 39.** η vs Frequency

Figure 40. $P_{REFL.}/P_{IN}$ vs Frequency

118 - 136 MHz Amplifier (see Figure 37)
Before Coil Adjustment

6. LITERATURE

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